Rules of Inference

1. Modus Ponens (M.P.)
   $p \to q$
   p
   $\therefore q$

2. Modus Tollens (M.T.)
   $p \to q$
   $\neg q$
   $\therefore \neg p$

3. Hypothetical Syllogism (H.S.)
   $p \to q$
   $q \to r$
   $\therefore p \to r$

4. Disjunctive Syllogism (D.S.)
   $p \vee q$
   $\neg p$
   $\therefore q$

5. Constructive Dilemma (C.D.)
   $(p \to q) \wedge (r \to s)$
   $p \vee r$
   $\therefore q \vee s$

6. Absorption (Abs.)
   $p \to q$
   $\therefore p \to (p \wedge q)$

7. Simplification (Simp.)
   $p \wedge q$
   $\therefore p$

8. Conjuction (Conj.)
   p
   q
   $\therefore p \wedge q$

9. Addition (Add.)
   p
   $\therefore p \vee q$

Replacement: Any of the following logically equivalent expressions can replace each other wherever they occur:

10.	De Morgan's Theorems (De M.):	$\neg(p \wedge q) \equiv (\neg p \vee \neg q)$
		$\neg(p \vee q) \equiv (\neg p \wedge \neg q)$
11.	Commutation (Com.):	$(p \vee q) \equiv (q \vee p)$
		$(p \wedge q) \equiv (q \wedge p)$
12.	Association (Assoc.):	$[p \vee (q \vee r)] \equiv [(p \vee q) \vee r]$
		$[p \wedge (q \wedge r)] \equiv [(p \wedge q) \wedge r]$
13.	Distribution (Dist.):	$[p \wedge (q \vee r)] \equiv [(p \wedge q) \vee (p \wedge r)]$
		$[p \vee (q \wedge r)] \equiv [(p \vee q) \wedge (p \vee r)]$
14.	Double Negation (D.N.):	$p \equiv \neg\neg p$
15.	Transposition (Trans.):	$(p \to q) \equiv (\neg q \to \neg p)$
16.	Material Implication (Impl.):	$(p \to q) \equiv (\neg p \vee q)$
17.	Material Equivalence (Equiv.):	$(p \equiv q) \equiv [(p \to q) \wedge (q \to p)]$
		$(p \equiv q) \equiv [(p \wedge q) \vee (\neg p \wedge \neg q)]$
18.	Exportation (Exp.):	$[(p \wedge q) \to r] \equiv [p \to (q \to r)]$
19.	Tautology (Taut.):	$p \equiv (p \vee p)$
		$p \equiv (p \wedge p)$